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# Spectral shape of cosmic rays over the Galaxy

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**Abstract.** In a recent paper we pointed out that there is fairly strong evidence for the intensity of cosmic ray positrons being lower than that expected from interactions of cosmic ray nuclei with interstellar matter using the simple 'leaky box' model. In the present work we examine more general models of propagation of Galactic cosmic ray nuclei and positrons and show that the discrepancy is present for other models too; only if the shape of the cosmic ray spectrum varies from place to place in the Galaxy does it appear possible to reconcile the positron data.

## 1. Introduction

The problem of the origin of cosmic rays is a continuing one. In previous papers the present authors and their colleagues have put forward evidence favouring a general Galactic origin in the range $10^9$–$10^{10}$ eV (Dodds *et al* 1975, Wolfendale and Worrall 1976) and for a group of particles in the range $10^{14}$–$10^{16}$ eV having come from specific Galactic sources: pulsars (Karakula *et al* 1974). The evidence in the range $10^9$–$10^{10}$ eV, which is the important energy region for the present work, comes from studies of $\gamma$ rays. Measured $\gamma$-ray intensities towards the Galactic anti-centre show that the cosmic ray intensity falls slowly as one proceeds outwards—and there is some support (but not as strong) for a corresponding increase in intensity as one proceeds inwards towards the Galactic centre (Stecker 1976, Dodds and Wolfendale 1976).

In the present work we examine the relevance of measurements of the energy spectrum of positrons to the origin problem.

## 2. The energy spectrum of positrons

In a recent paper (Giler *et al* 1977, to be referred to as I) the present authors calculated the expected energy spectrum of positrons arising from the interaction of cosmic ray nuclei with the interstellar medium in the following way. Taking the local spectrum of nuclei, corrected for solar modulation, and using this together with the assumed composition of the interstellar medium (ISM) the production rate of positrons per gram of ISM was calculated. The ratio of the observed positron flux to the calculated production rate per gram was then taken as an 'effective grammage' for positron collection. This grammage can be referred to as the 'apparent grammage' for positrons but it should be pointed out that it is not necessarily the actual grammage traversed by positrons and, in fact, in most of the propagation models it is not.

It was shown in I that the apparent grammage for positrons was less than that given for nuclei; for example at 5 GeV positron energy the grammage needed for positrons is $3 \pm 1$ g cm$^{-2}$, compared with the 'measured' value of around 7 g cm$^{-2}$ for nuclei.

At this stage it should be remarked that the grammage for nuclei is derived in a different way: an assumed mass composition of nuclei at production is followed through a sufficient mass of ISM to give the locally measured isotopic composition. Insofar as different groups of nuclei appear to propagate in the same fashion, from $\alpha$ particles through to iron, it is assumed that the same grammage applies to protons; it is necessary to bear in mind that this is an assumption, however, although a very reasonable one. In what follows the cosmic ray nuclei are referred to simply as 'protons'.

In I we indicated, briefly, a number of ways of explaining the difference in the grammages. In the present work attention is restricted to diffusion models.

The treatment does not pretend to be exhaustive and, indeed, one-dimensional diffusion alone is examined, in some cases, but the analysis brings out the basic features.

## 3. Propagation of nuclei and positrons

### 3.1. *Energy-independent propagation*

*3.1.1. The leaky-box model.* It is easy to show that in a simple homogeneous model ('leaky box' type model) the intensity of positrons should be proportional to their production rate based on the local proton spectrum and to their residence time. The latter should be identical with that for protons of the same rigidity. If the residence time is a function of energy then the residence time for the positrons of energy under consideration is still the operative quantity—the residence time for the proton parents of the considered positrons (i.e. of energy $\sim 10\,E_{e^+}$) is not relevant because it affects the datum local proton intensity too.

*3.1.2. Cosmic rays from a single source.* After a homogeneous situation the next simplest is where there is a single continuous source of protons and these particles propagate in one dimension. Such a model might conceivably be relevant to the actual state of affairs if, for example, cosmic rays were produced only at the centre of the Galaxy and they were constrained to move along the spiral arms. A more realistic model can be built up by having specific sources (supernovae?) distributed along the arms. In both cases escape out of the arms is allowed.

Consider a source of cosmic rays at $x = 0$ in a one-dimensional situation, the particles being able to undergo 'isotropic' diffusion with diffusion coefficient $D$ and mean lifetime against escape $T$. At present $D$ and $T$ are assumed independent of energy. The proton flux at $x$ is

$$n_{\rm p}(x) = \frac{q}{2(\pi D)^{1/2}} \int_0^\infty \exp\left(-\frac{t}{T}\right) \frac{1}{t^{1/2}} \exp\left(-\frac{x^2}{4Dt}\right) \mathrm{d}t$$

which leads to

$$n_{\rm p}(x) = \frac{q}{2}\left(\frac{T}{D}\right)^{1/2} \exp\left(-\frac{|x|}{(DT)^{1/2}}\right) \qquad (1)$$

where $q$ is the production rate at the source. The mean life of the protons recorded at $x$ is

$$\bar{t}_p(x)=\frac{T}{2}\left(\frac{|x|}{(DT)^{1/2}}+1\right). \qquad (2)$$

If the generation rate of positrons is $K$ (per 'average' proton per unit time), and if positrons and protons propagate in an identical fashion, then the positron flux at $x$ is, in the absence of energy loss:

$$n_e(x)=\frac{qKT^2}{4(DT)^{1/2}}\left(\frac{|x|}{(DT)^{1/2}}+1\right)\exp\left(-\frac{|x|}{(DT)^{1/2}}\right) \qquad (3)$$

that is,

$$n_e(x)=Kn_p(x)\bar{t}_p(x). \qquad (4)$$

Thus, the positron flux should be equal to the product of the local proton flux, the mean life of protons and the rate of production (per unit time) of positrons per 'average' proton. This means that the apparent grammage for positrons found in the usual way should be the same as that for protons. (The grammage is simply the mean lifetime times the average gas density and velocity.) Although such a result is obvious in the general diffusive situation, as was pointed out in § 3.1.1, where one is well away from specific sources, it is less obvious here.

The situation with the actual mean life of the detected positrons is different. The mean life of the protons is given by equation (2) but that for positrons can be shown to be

$$\bar{t}_e(x)=\frac{T}{4}\left(\frac{Z^2}{Z+1}+3\right)$$

where $Z$ is $|x|/(DT)^{1/2}$. Thus $\bar{t}_e(x)$ is greater than $\bar{t}_p(x)$ for small values of $|x|/(DT)^{1/2}$ and smaller for large $|x|/(DT)^{1/2}$ (the times are equal for $|x|/(DT)^{1/2}=0{\cdot}62$).

In the situation considered here, the magnitude of the flux of protons is clearly a function of $x$ although, because energy-independent propagation is assumed, the spectral shape will be the same throughout. Despite the change in proton flux the apparent positron grammage is unaltered. This is clearly true whatever the distribution of sources. However, if a number of sources are present which have different spectral shapes then the apparent grammage will change.

### *3.2. Energy-dependent diffusion*

*3.2.1. Constant spectral shape everywhere.* Inspection of equation (1) shows that the spectral shape will remain constant, independent of $x$, if the product $D(E)T(E)$ is independent of energy. $D(E)$ and $T(E)$ may be separately energy dependent and indeed there is strong experimental evidence that they are, in practice, in view of the fact that the nuclear grammage is found to vary with $E$.

Insofar as the spectral shape of cosmic ray nuclei is usually considered to be invariant this situation requires further analysis. Let $T(E)D(E)=T_0D_0=\text{constant}$, where $T_0$ and $D_0$ are the values corresponding to the energy of positrons to be considered, $E_0$. The positron flux will be

$$n_e(x,E_0)=\frac{Kq(E_1)}{4}\frac{T_1(E_1)}{(T_0D_0)^{1/2}}\left(\frac{T_0}{D_0}\right)^{1/2}\int_{-\infty}^{\infty}\exp\left(-\frac{|u-x|}{(D_0T_0)^{1/2}}\right)\exp\left(-\frac{|u|}{(D_0T_0)^{1/2}}\right)\mathrm{d}u \quad (6)$$

where $q(E_1)$ is the production rate at the source of protons of energy $E_1$ which can be considered to give rise to positrons of energy $E_0$ ($E_1/E_0 \simeq 10$). The quantity $K$ is the number of positrons of energy $E_0$ produced by one proton of energy $E_1$ per unit time. $K$ is found by integrating the production cross section over the energy spectrum in the standard way; it will, in fact, be a slowly varying function of the spectral shape but this factor is neglected:

$$n_e(x, E_0) = \frac{Kq(E_1)}{2}\left(\frac{T_1}{D_1}\right)^{1/2}\frac{T_0}{2}\left(\frac{|x|}{(D_0T_0)^{1/2}}+1\right)\exp\left(-\frac{|x|}{(D_1T_1)^{1/2}}\right) \tag{7}$$

$$n_e(x, E_0) = K\bar{t}_p(E_0)n_p(x, E_1). \tag{8}$$

This equation is similar to equation (4) but it is to be noted that the flux of positrons of energy $E_0$ is proportional to the grammage of protons of the same energy $E_0$ (or, more precisely, of the same rigidity) rather than the grammage of the 'actual' parents.

The conclusion is thus that the discrepancy between the proton and positron grammages at the same rigidity cannot be resolved by varying $D(E)$ and $T(E)$ unless the product $D(E)T(E)$ also varies, i.e. unless there is a dependence of spectral shape on position in the Galaxy.

*3.2.2. Spectral shape varying over the Galaxy: constant lifetime.* The simplest case is to assume that either $D$ or $T$ is independent of energy and to chose the energy dependence of the other so that the measured form of $t_p(E)$ is reproduced. Here we examine the situation where $T$ is constant; $D(E)$ then follows, using equation (2), as

$$D(E) = \frac{x^2T}{(2\bar{t}_p(E)-T)^2}. \tag{9}$$

The corresponding form of the energy spectrum as a function of distance is given by

$$n_p(x, E) = \frac{q(E)}{2}\left(\frac{T}{D(E)}\right)^{1/2}\exp\left(-\frac{|x|}{(D(E)T)^{1/2}}\right). \tag{10}$$

The positron flux can be calculated in a similar manner to that considered earlier and is:

$$n_e(x, E_0) = \frac{Kq(E_1)}{2}T^{3/2}\left[\frac{D_1^{1/2}}{D_1-D_0}\exp\left(-\frac{|x|}{(D_1T)^{1/2}}\right)-\frac{D_0^{1/2}}{D_1-D_0}\exp\left(-\frac{|x|}{(D_0T)^{1/2}}\right)\right]. \tag{11}$$

In this expression, $D_1$ is the diffusion coefficient of the parent protons of energy $E_1$ and $D_0$ that of the positrons of energy $E_0$.

It can be seen that equation (11) reduces to equation (3) in the limit as $D_1$ tends to $D_0$.

A useful form follows for the situation where $D_1 \gg D_0$:

$$n_e(x, E_0) \simeq \frac{Kq(E_1)}{2}\left(\frac{T}{D_1}\right)^{1/2}\frac{TD_1}{D_1-D_0}\exp\left(-\frac{|x|}{(D_1T)^{1/2}}\right) \tag{12}$$

from which

$$n_e(x, E_0) \simeq K\frac{TD_1}{D_1-D_0}n_p(x, E_1) \tag{13}$$

(remembering that this relation is only valid for $D_1 \gg D_0$). The positron intensity is now reduced compared with its value for no cosmic ray spectral gradient by a factor

$$F \simeq \frac{2D_1(D_0T)^{1/2}}{(D_1-D_0)[|x|+(D_0T)^{1/2}]}. \tag{14}$$

This factor is thus the ratio of the apparent lifetime of the positrons to that of the protons of the same rigidity.

As an example of the application of this idea calculations have been made for specific values of the parameters. A value of $x = 10$ kpc was taken as being illustrative of the linear dimensions. $D(E)$ was chosen to give the 'observed' dependence of $\lambda_N$ on energy (figure 1) using a mean ISM gas density of 0·1 atom cm$^{-3}$ and $T = 4{\cdot}8 \times 10^{14}$ s (to fit the $^{10}$Be data). The form of $D(E)$ is given in figure 2 and the corresponding variation of $\lambda$ with energy is shown in figure 3. It is apparent that this choice of parameters gives $\lambda$ at least in the experimental region.

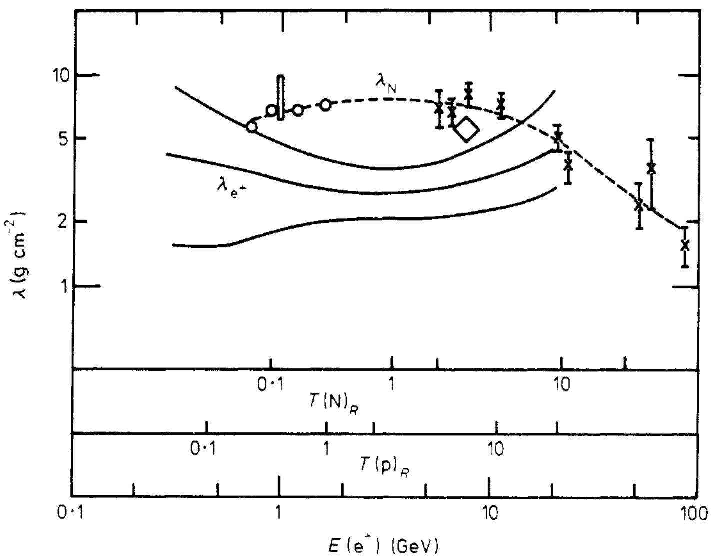


**Figure 1.** Mean 'grammage' against energy. $\lambda_N$ refers to nuclei; the source of the experimental values is described in I. The curve marked $\lambda_{e^+}$ is the best estimate of the 'grammage' for positrons and its derivation is given in I; the bracketing curves represent one standard deviation limits.

$T(N)_R$ and $T(p)_R$ are kinetic energies of nuclei and protons of the same rigidity as positrons of the energies indicated.

Although the present one-dimensional treatment may not be completely realistic it is profitable to study the associated gradient of the cosmic ray spectrum. Figure 4 gives the spectrum as a function of $x$ along the axis (spiral arm). It is seen at once that the gradient is rather large; for example, at 5 kpc towards the source, a distance characteristic of the scale over which the bulk of the detected particles come, the intensity of protons of a few GeV is about twenty times that locally.

*3.2.3. Spectral shape varying over the Galaxy: variable lifetime.* In the general case both $D$ and $T$ will be functions of energy and calculations have been made for this situation too.

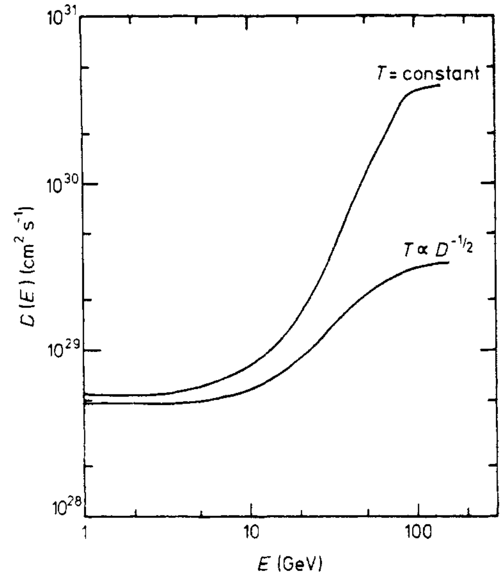


**Figure 2.** Alternative forms for the variation of diffusion coefficient $D(E)$ with energy. See §§ 3.2.2 and 3.2.3 for the reasons for values shown.

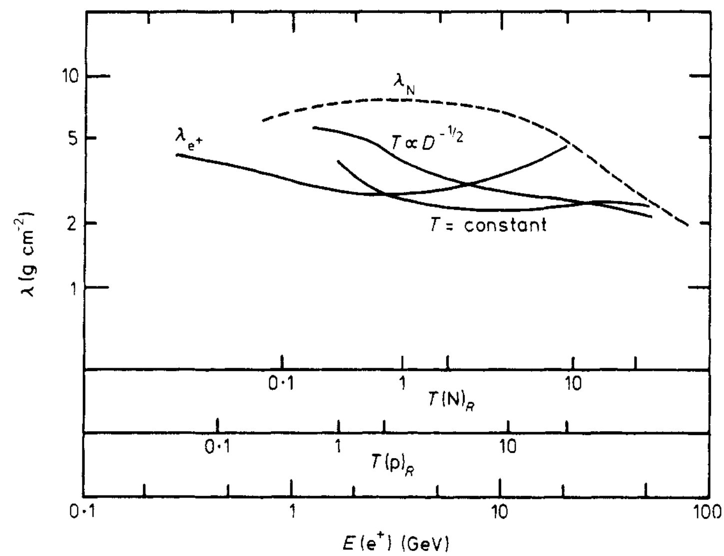


**Figure 3.** Mean 'grammage" against positron energy (and nucleus and positron kinetic energy—see caption to figure 1). $\lambda_N$ relates to nuclei. $\lambda_{e^+}$ is the best experimental curve of figure 1 and refers to positrons. $T$ = constant and $T \propto D^{-1/2}$ relate to predictions for the positron grammage for the alternative cases considered in the text.

The general expression for the reduction factor is now:

$$F = \frac{2(D_1T_1D_0T_0)^{1/2}}{(D_1T_1 - D_0T_0)[|x| + (D_0T_0)^{1/2}]}[(D_1T_1)^{1/2} - (D_0T_0)^{1/2}\exp(-\mu x)] \tag{15}$$

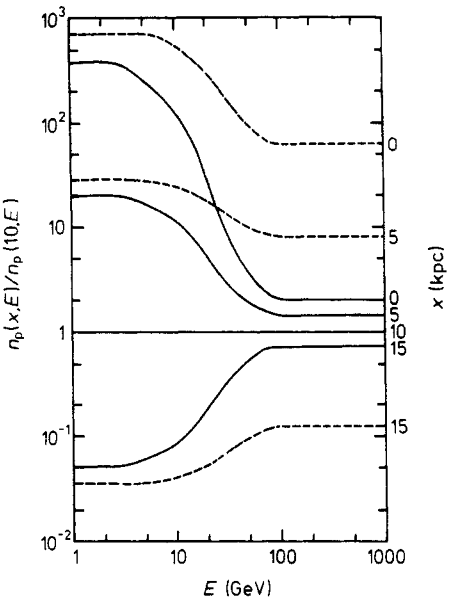


**Figure 4.** Ratio of the proton intensity at a distance $x$ from the source to that at 10 kpc (i.e. the local value).

The full curves refer to the case where $T$ = constant and the broken curves to $T \propto D^{-1/2}$ (figure 2).

where

$$\mu = \frac{(D_1T_1)^{1/2} - (D_0T_0)^{1/2}}{(D_1T_1D_0T_0)^{1/2}}$$

(this reduces to (14) if $T_0 = T_1$ and $D_1 \gg D_0$).

The number of possible combinations of $D(E)$ and $T(E)$ is of course infinite; here we simply try $D(E) \propto T(E)^{-2}$ and values of $D(E)$ and $T(E)$ have been chosen to give the $\lambda$ dependence, as before. Calculations have been made for $T_0 = 4{\cdot}2 \times 10^{14}$ s and $D_0 = 4{\cdot}8 \times 10^{28}$ cm$^2$ s$^{-1}$ with the result shown in figure 3 (the variation of $D$ with $E$ is given in figure 2). The dependence of proton intensity on distance is given in figure 4.

It is seen that although the dependence of diffusion coefficient on energy is less rapid than for the case when $T$ was independent of $E$, the gradient of proton intensity is even higher.

## 4. Discussion

### *4.1. Validity of the simple models*

It has been pointed out that the simple one-dimensional models may be more useful than appears at first sight if diffusion is largely limited to directions along the spiral arms. However, the models considered above have serious drawbacks which relate to inconsistencies with other data. Firstly, there are problems with the very large cosmic ray gradients predicted. Although the significant anisotropies in arrival directions which will be predicted will not necessarily be inconsistent with experiment, because of the smearing effect of the interplanetary field at the energies in question, there are difficulties with the predicted $\gamma$-ray fluxes. These $\gamma$ rays are generated in few GeV-proton–ISM collisions and, as has been shown elsewhere, the Galactic $\gamma$-ray measurements can be used to study the product of proton intensity and gas density in particular regions of the Galaxy. Such high proton intensities, albeit in remote and perhaps small regions, would appear to give $\gamma$-ray intensities which would have been seen.

The second problem concerns the predicted distributions of proton (nucleus) path lengths, which are narrower than inferred from the experimental data. The sharpness of the predictions arises from the fact that only one source has been assumed.

The conclusion to be drawn at this stage is that we have demonstrated that the positron data can be reconciled with information on the spectra of protons and heavier nuclei by requiring that the shape of the cosmic ray spectrum varies from place to place in the Galaxy because of energy-dependent propagation (it is implicitly assumed that the nuclei are all produced in the Galaxy). A model can be chosen with a single source of nuclei which will give the observed positron intensity but it is unlikely that it will give consistent data with other cosmic ray observations. It is much more likely that a generally consistent picture can be built up with a more sophisticated model involving many sources and such a model will be examined in a later publication.

At this stage it is profitable to examine the likelihood, from other evidence, of variations of cosmic ray spectra across the Galaxy.

### *4.2. Likelihood of variable cosmic ray spectrum*

In the absence of firm evidence about specific cosmic ray sources a discussion of spectral variability is necessarily speculative. However, some general remarks can be made.

The likelihood of variations in the cosmic ray intensity, irrespective of shape, have already been mentioned. The work of Dodds and Wolfendale (1976) on $\gamma$ rays from the direction of the Galactic anti-centre appears to indicate a gradual fall-off with increasing Galactocentric distance. French and Osborne (1976) have found evidence from an analysis of synchrotron radiation for a rather large ratio between the emissivity in the spiral arms and in the interarm region together with a large scale Galactocentric radius variation. The radiation arises from the motion of cosmic ray electrons in the interstellar magnetic fields and it is our contention that some measure of coupling should exist between the electron intensity and the mean square field so that we feel that a likely interpretation of the results is that there are variations in space of the intensity of electrons.

There is also some evidence from $\gamma$-ray data for a gradient of the intensity of cosmic rays of a few GeV along the local spiral arm. Osborne *et al* (1976) studied the SAS II $\gamma$-ray data together with the information from neutral hydrogen measurements to show that the proton intensity was probably some 50% higher in a region 2 kpc from the earth towards $l^{\mathrm{II}} \simeq 270°$. They showed that this result was consistent with the apparent anisotropy of protons of several hundred GeV from the work of Marsden *et al* (1976). Although the gradient is too small to be consistent with the analysis made in § 3.2.2 it does demonstrate that gradients are likely and that the present ideas have promise.

It is interesting also to investigate the cosmic ray intensity at the earth as a function of time over very long periods; insofar as the solar system moves through the Galaxy, rotating in about $2\times10^8$ yr, it has sampled different regions. It appears that the solar system oscillates about the Galactic plane with an amplitude of around 80 pc and rotates such that its variation in Galactocentric radius over a rotation is of the order of $\pm0{\cdot}5$ kpc (Mihalas and Routly 1968). Thus, in a rotation of the Galaxy the sun should penetrate a number of arm and interarm regions.

As yet, the experimental data on the long term variations of the flux are somewhat scanty. Shaeffer (1975) has recently surveyed the situation and his conclusion is that the cosmic ray intensity has not varied by more than a factor of two over the past $10^9$ years (by 'cosmic ray intensity' is meant the intensity above several hundred MeV). Schaeffer also concludes that the intensity averaged over the past $4\times10^5$ years is 50% higher than that averaged over the past $10^9$ years and argues that this is evidence for a lower intensity some distance above and below the Galactic plane. However, we consider that the same result would follow if the sun is now in a weak spiral arm where the cosmic ray intensity is higher than in the interarm region—the sun having spent a large fraction of its time in the interarm region in the past $10^9$ years. It can be concluded then that the long term intensity variations would be consistent with a high arm/interarm gradient but in view of the fact that the range of Galactocentric distance 'scanned' is small, radial gradients cannot be studied. Large gradients could, in fact, be present—in random fashion along the arms—but these would be unlikely to show up in the integral time exposures contributing to the result. Concerning short period gradients, Dergachev and Malchenko (1974) and Kocharov *et al* (1975) have given evidence from studies of sea sediment cores for an increase in the cosmic ray flux by a factor of about 5 some $2\times10^6$ years ago for a period of perhaps $10^6$ years, but although this evidence would support our model it cannot be regarded as strong.

## 5. Conclusions

If the analysis reported in I is correct, i.e. if the flux of cosmic ray positrons is smaller than expected on the basis of an energy-independent diffusion model, then a model involving energy-dependent diffusion seems to offer a convincing explanation. Such a model must imply a spectral shape of protons which varies from place to place in the Galaxy.

We regard the acquisition of better experimental data on primary positrons as being an important step towards elucidating the problem of the origin of cosmic rays. If the experimental results reported so far are correct then we regard this as comprising quite strong evidence favouring Galactic origin at the energies in question.

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